AD/RHIC/AP-85/Revised

Analytic Evaluation of Tune Shift Due to Octupole Correctors

M.J. Rhoades-Brown

BNL

September 12, 1989

The tune shifts due to the octupole correctors are tabulated below. They were requested by G. Raka as part of his study on transverse instabilities in RHIC. The tune shifts were calculated with an analytic formula due to Jackson[1]. In this formalism the horizontal component of the octupole tune shift is given by,

$$\delta\nu_x^{(3)} = \frac{3}{2} b_3 N \delta^2 \langle \beta_x X^2 \rangle + \frac{\epsilon_x}{4} \langle \beta^2 \rangle \tag{1}$$

where δ is the momentum deviation, N is the number of octupole correctors in the ring (138), b_3 is the multipole coefficient in dipole units, ϵ_x is the horizontal emittance divided by π, and the averages $\langle\ \rangle$ are defined for a single octupole corrector by

$$\langle \beta_x X_p^2 \rangle = \frac{L}{2\pi\rho} \frac{1}{2} (\beta_x X_p^2 |_F + \beta_x X_p^2 |_D) \tag{2}$$

$$\langle \beta^2 \rangle_x = \frac{L}{2\pi\rho} \frac{1}{2} (\beta_x^2 |_F + \beta_x^2 |_D) \tag{3}$$

L is the length of a single octupole corrector (.5m), and ρ is the dipole bending radius (243.241m). For RHIC, the lattice functions $\beta_{x\,F}$ & $\beta_{X\,D}$ have the values 49.73m & 9.04m, and $X_{p\,F}$ & $X_{p\,D}$ have the values 1.59m & .78m respectively. With these values (1) becomes

$$\delta\nu_x^{(3)} = b_3 (5.61\delta^2 + 27.30\epsilon_x) \tag{4}$$

The integrated strength of the octupole correctors is quoted[2] as $B'''L=3600T/m^2$. Utilizing the relation between B''' and the Taylor series expansion coefficient b_3, $B'''=n!B_o b_n$, it is found

$$b_3 = \frac{B'''}{6B_o} = \frac{1.2\times10^3}{B_o} \frac{T}{m^3} \tag{5}$$

where B_o is the dipole field.

Table I

Energy	10.4	30.0	100.00
$\delta(\times 10^{-4})$	±10.2	±8.3	±3.1
ϵ_x^N πmm mrad	10	10	10
ϵ_x πmm mrad	.82	.3	.093
B_o T	.35	1.15	3.45
$\Delta\nu_x$ $(\times 10^{-3})$	96.76	12.58	1.07

References

(1) A. Jackson, "Tune Shifts and Compensation from Systematic Field Components", SSC Report No. SSC-107.

(2) Conceptual Design of Relativistic Heavy Ion Collider RHIC, May 1989, BNL52195.